Foreword

Hello! Despite how the following paper is written, it is in fact written by me, the Gorilla of Destiny. I just thought it would be more fun to frame this as internal drama in the academic world of theoretical magic. One thing I really want to emphasise is that **I MIGHT BE WRONG**. Literally everything in this paper might be incorrect, there may be no flaw in the theory of magic, just my understanding, and the goal of this paper is to help you either spot my flaw, or (in the unlikely case where I have made no mistakes) understand why we must alter some of the theory of magic. In the paper I actually highlight a few areas where I think I've made a mistake. This is far from my most meticulous work as much of it was done/written in the final weeks of me writing my PhD thesis, so as you can imagine my time and mind were often elsewhere. So expect errors, sentences that are wonky, grammar and spelling mistakes, and even sentences that don't conclu-.

This is a good time though to underline a key theme of my works. We are doing 5e *SCIENCE*. Science is a way of approaching problems and sometimes your models have mistakes, or errors, or oversights. In fact, there are cases in the real world that have flaws similar to this one (e.g. the ultraviolet catastrophe). The discovery of this potential flaw is not a bad thing at all, and actually, I think it is very exciting. I also want to highlight that we are ***not*** in the business of merging real physics and 5e magic. We are not trying to make magic adhere to rules of physics it most certainly violates. We are building a model to describe the phenomena we see in the rules, and as such we are not constrained to real physics. As a rule of thumb, if you start including any real-world physical constants, you've probably started adding things you shouldn't have. It's tough to do, especially if you are a physicist, but it is vital to keep in mind.

Final thing, if you have any suggestions or comments, feel free to ping me on discord or email me at gorillaofdestinytiktok@gmail.com. I might miss it but I will try my bestest not to.

Have a lovely day, cheers!
- The Gorilla of Destiny.

Archmage of the University of Theoretical Magic (and all round idiot)

P.S. I have used the names Preservationist and Aedificarist for the two camps of magic theorists in this faux debate, but if they are the names of real groups I am sorry and don't want to lend any credence to them should they exist. A cursory google doesn't return anything but you never know.

To Infinity and Beyond: Fatal Flaws in the Existing Theory of Magic

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Faroe Foundson

Abstract

The Theory of Magic, as presented by the Gorilla of Destiny, contains a fatal flaw. Recent work has shown that in order to function as described, in both classical and quantum cases, the theory demands infinite energy from the magic-field. This seems unlikely and thus underlines a flaw in the theory of magic. Two groups of research have thus emerged, the Preservationists, who aim to preserve as much of the original theory as possible, and the Aedificarists, who build new models for magic. Here we review the leading theories from both camps and summarise their findings such that a reader might understand each argument. While this work is ongoing, it is vital we begin to categorize and develop each of these theories.

1 Introduction

The impact of the formative work done by The Gorilla of Destiny, archmage of the University of Theoretical magic, cannot be denied. The Theory of Magic [3] forms the foundation on which modern understanding, practice, and technology has been built [8, 2]. However, some researchers are aware that due to the revolutionary nature of this work, it has not received the same critical treatment as we might expect of the leading theories within other fields of natural philosophy. In this paper, we will detail a key flaw in the current formulation of the so-called "Theory of Magic", and identify and review the now emerging solutions and camps within the field. In order to adequately explain the flaws within the theory, we must first explain the theory as proposed in the original paper, and the more modern approach proposed in recent research.

1.1 The Theory of Magic

The Theory of Magic has two main running theories, the classical and the quantum. First, in section 1.1.1, we will focus on the classical view, as this was the original proposed theory in [3]. However, recent theories have attempted to explain the original theory in a quantum frame-work, which we will detail in section 1.1.2.

Both theories contain similarities. In both, magic begins with some sort of caster[1], which acts on the well of magic energy, commonly known as the weave, though notably this may or may not be distinct from the magical-field on which spells act. A caster inputs some amount of energy, which then causes the quasi-stable well to collapse resulting in exponentially greater returns in energy. This energy is then redirected by the caster in order to create a spell on the magic-field. Current debate surrounding the nature of this "well" is ongoing with no clear answer given and further research required**CHECK ToM FOR THIS**. Similarly, to account for the varying volume/area of effect a spell may have, all iterations of magic theory use the "Aperture Hypothesis". This describes the impact volume of a spell as a uniform application of the spell from the magic-field to a given area controlled by the magic user. Though, current research suggests certain spells prefer shapes, giving rise to the standards we see across cultures and classes [5].

While these aspects of the theory remain hypotheses and have a significant number of outlying questions that further research must answer, we will instead focus on the core of the theory, which explains how spells move in the magic field.

1.1.1 Classical View

The classical model of a spell proposes that the magic-field is analogous to an elastic sheet. A spell is then a two-dimensional bump in this sheet, with one dimension being the damage (δ) dimension[2], and the other being the spatial direction. By convention we say that the spell moves in a one dimensional line x, where $x = 0$ is the caster, $x = T$ is the target, and $x = R$ is the maximum

[1] commonly a creature of some kind but as shown in [6], this is not necessarily the case.

[2] In some spells this is actually a vector existing between or along two or more orthogonal damage dimensions.

range of the spell. The form of this bump is then given by:

$$f(\delta, x, t) = A \times \underbrace{\exp - \frac{(\delta - \mu_D)^2}{2\sigma_D^2}}_{\text{Damage}} \times \underbrace{\exp - \frac{(x - \mu_x(t))^2}{2\sigma_x(t)^2}}_{\text{Space}} \quad (1)$$

where A is a constant, μ_D and σ_D are the mean and standard deviation of the damage distribution, and $\mu_x(t)$ and $\sigma_x(t)$ are the mean and standard deviation of the spell in the spatial direction, with respect to time. We define $\mu_x(t)$ and $\sigma_x(t)$ as:

$$\mu_x(t) = vt, \quad (2)$$

$$\sigma_x(t) = \lambda_0(1 - \frac{v}{R}t), \quad (3)$$

with λ_0 a constant (which we will return to shortly). This definition of $\mu_x(t)$ simply states that spells move with a constant velocity v from $x = 0$ along the positive direction. The definition of $\sigma_x(t)$ means that at $x = R$ (which occurs when $vt = R$, and thus $\frac{vt}{R} = 1$) $\sigma_x(t) = 0$, meaning the spell's width in the spatial direction is 0 and thus it vanishes.

In [8], the authors suggest that λ_0 is the initial energy of the spell, which diminishes as the spell moves along the x-axis. This results in the light we see as spells move from caster to target. When the spell reaches its target, the damage and position are then chosen at random from this distribution. To get the probability density, we therefore normalize the equation such that:

$$\int_{-\infty}^{\infty} \int_{-\infty}^{\infty} P(\delta, x) dx d\delta = 1 \quad \forall\, t, \quad (4)$$

essentially saying "The spell must give an output of a real damage amount and real spatial position". This can be done using (derivation shown in section A):

$$A = \frac{1}{\sqrt{2\pi}\sigma_D} \times \frac{1}{\sqrt{2\pi}\sigma_x(t)} \quad (5)$$

$$A = \frac{1}{2\pi\sigma_D\sigma_x(t)}. \quad (6)$$

With this, we thus get the behaviour of spells as seen in experiments.

[3] *Really, the real life actual one*

1.1.2 Quantum View

To take the above concepts further, we do not take the spell as a deformation of an elastic magic-field, but instead as a wave. This borrows from the idea of "Quantum Mechanics"[3], wherein incredibly small particles can be described by a wave. When observed, the wave-function collapses, providing a single measured observation. For example, let us imagine that there are two possible states for a particle to be in, we will call these $|0\rangle$ and $|1\rangle$. What these are doesn't matter, but we will say for this that these two states exist orthogonally to each other. We can then describe something as existing as both of these states simultaneously:

$$|\psi\rangle = \frac{1}{\sqrt{2}}|0\rangle + \frac{1}{\sqrt{2}}|1\rangle. \quad (7)$$

The probability of the wave function collapsing into a given state is given by the square of the coefficient before the state. So in eq. (7), we are saying that both states $|0\rangle$ and $|1\rangle$ are equally likely. One condition of this is that the probabilities sum to 1, as with a normal probability density.

For wave-functions which are continuous, we can obtain the probability density from $|\psi|^2$, this can be obtained by multiplying the wave function with its complex conjugate. The complex conjugate of any number or function is a reflection of said number or function on the real axis, that is to say, we invert the sign in-front of every complex number. So a complex number $z = a + ib$ becomes $z^* = a - ib$ and vice versa. Similarly, a complex function $z(x) = e^{-i\theta} \to z^*(x) = e^{i\theta}$.

For magic theory, we therefore can propose the quantum magic wavefunction:

$$\psi(\delta, x, t) = \underbrace{\left(\frac{1}{2\pi\sigma_D\sigma_x(t)} e^{-\frac{(\delta-\mu_D)^2}{2\sigma_D^2}} e^{-\frac{(x-\mu_x(t))^2}{2\sigma_x(t)^2}} \right)^{0.5}}_{\text{Probability}} \times \underbrace{e^{i(kx-\omega t)}}_{\text{Wave}} \quad (8)$$

Where k is the wave-number of the wave ($2\pi/\lambda$, with λ the wavelength), and ω the angular frequency of the wave. Here we can see that squaring the probability in front of the wave returns the probability density as before. In practice, this means we are getting the same results as before, but with a different model of the magic field. In the classical view, the spell is a perturbation, in the quantum it is a wave. For details on why $\exp -i(kx - \omega t)$ is a wave, see section B. Now that we have built these models, it is time to tear them down.

2 Infinite Potential Energy: Classical Flaw

When an elastic sheet is stretched, its potential energy is increased. As an example, we can imagine a spring. If we pull the spring by applying some force to it, it now has an increased potential energy. If we remove the stretching force, the spring converts this to kinetic energy and returns to its unstretched position. In essence, the potential energy is the work done by the force of the spring trying to retract to its original position. The amount of force used to stretch a spring by ΔX (and hence, the force exerted in return by the spring) is given by:

$$F = \kappa \Delta x, \tag{9}$$

where κ is the spring constant (typically we might use k, but since that is already being used for wave-number, we will elect to use κ). The potential energy of the spring is then the work done by the restoring force F, which is defined by the integral:

$$\begin{aligned} U &= \int_{-\infty}^{\infty} F dx && (10) \\ &= \int_0^{\Delta x} \kappa x dx && (11) \\ &= \left[\frac{1}{2}\kappa x^2\right]_0^{\Delta x} && (12) \\ &= \frac{1}{2}\kappa \Delta x^2 && (13) \end{aligned}$$

For our elastic sheet, this generalises to (section C):

$$U_{\text{total}} = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \frac{1}{8}\kappa \left(\left(\frac{\partial f}{\partial x}\right)^2 + \left(\frac{\partial f}{\partial \delta}\right)^2\right) dx d\delta, \tag{14}$$

$$U_{\text{total}} = \frac{1}{8}\kappa \left(\int_{-\infty}^{\infty} \left(\frac{\partial f}{\partial x}\right)^2 dx + \int_{-\infty}^{\infty} \left(\frac{\partial f}{\partial \delta}\right)^2 d\delta\right) \tag{15}$$

assuming that the stretch in x and δ are independent, also κ here is slightly different from κ in the spring equation but for this, what matters is it is a constant determining the ease at which the fabric can stretch. If we consider *only* the x-direction:

$$\begin{aligned} \frac{\partial f}{\partial x} &= -\frac{x - \mu_x(t)}{\sigma(t)^2} f, && (16) \\ \Rightarrow U_{\text{total}} &= \frac{\kappa}{8}\int_{-\infty}^{\infty} \left(\frac{x - \mu_x(t)}{\sigma_x^2(t)}\right)^2 f^2 dx, && (17) \\ &= \frac{\kappa}{8}\int_{-\infty}^{\infty} \frac{1}{2\pi\sigma_x^2(t)} \frac{(x-\mu_x(t))^2}{\sigma_x^4(t)} \exp\left(-\frac{(x-\mu_x(t))^2}{\sigma_x(t)^2}\right) dx, && (18) \\ &= \frac{\kappa}{8}\frac{1}{2\pi\sigma_x^4(t)}\int_{-\infty}^{\infty} \frac{(x-\mu_x(t))^2}{\sigma_x^2(t)} \exp\left(-\frac{(x-\mu_x(t))^2}{\sigma_x(t)^2}\right) dx, && (19) \end{aligned}$$

$$\begin{aligned} \text{Let } u &= \frac{x - \mu_x(t)}{\sigma(t)} && (20) \\ \Rightarrow U_{\text{total}} &= \frac{\kappa}{8}\frac{1}{2\pi\sigma_x^4(t)}\int_{-\infty}^{\infty} u^2 e^{-u^2}\sigma_x(t) du, && (21) \\ \Rightarrow U_{\text{total}} &= \frac{\kappa}{8}\frac{1}{2\pi\sigma_x^3(t)}\frac{\sqrt{\pi}}{2} && (22) \\ \Rightarrow U_{\text{total}} &= \frac{\kappa}{32\sqrt{\pi}\sigma_x^3(t)} && (23) \end{aligned}$$

as $x \to R$, $\sigma_x(t) \to 0$, thus $\frac{\partial f}{\partial x} \to \infty$, therefore $U \to \infty$. This is not allowable in a physical system, as it is saying for a spell to reach its stated range, it requires infinite energy and will then also release infinite energy as the spell vanishes at $x = R$. This occurs regardless of whether we normalise g or not probabilistically[4].

This is a fundamental flaw in the classical view of the theory of magic, though there are potentially practical and theoretical solutions which we will discuss in

[4] *Author's Note: To me this is counter-intuitive and I suspect an error has been made here, if anyone can spot it let me know. To me it seems like if the stretching is not normalised (and hence not growing taller), then the potential should shrink as the bump gets thinner, in which case the Theory of Magic can survive almost entirely as is. Another reason to suspect something has gone wrong in section C, is that the formula for the energy does not reduce back to the spring example, as I might expect (basically, that the* $\frac{1}{8}$ *makes me suspect myself of stupidity).*

section 4. First, however, we will discuss a similar issue with the quantum approach to the magic theory.

3 Infinite Kinetic Energy: Quantum Flaw

For the elastic sheet, we considered the potential energy of the stretched elastic sheet, for the quantum approach we will instead consider the kinetic energy. The kinetic energy of a wave function is given by:

$$E_k = \alpha \int_{-\infty}^{\infty} \left| \frac{\partial \psi}{\partial x} \right|^2 dx, \tag{24}$$

with α some constant. In the x-direction, we can see our wavefunction as:

$$\psi_x = \sqrt{g_x} e^{i(kx - \omega t)}, \tag{25}$$

with g_x the Gaussian in the x-direction. Therefore:

$$\frac{\partial \psi}{\partial x} = \left(-\frac{x - \mu(t)}{\sigma_x^2(t)} + ik \right) \psi. \tag{26}$$

$$\Rightarrow \left| \frac{\partial \psi}{\partial x} \right|^2 = \frac{\partial \psi}{\partial x} \frac{\partial \psi^*}{\partial x}, \tag{27}$$

$$= \left(\left(\frac{x - \mu(t)}{\sigma_x^2(t)} \right)^2 + k^2 \right) |\psi|^2. \tag{28}$$

$$\Rightarrow \alpha \int_{-\infty}^{\infty} \left| \frac{\partial \psi}{\partial x} \right|^2 dx = \alpha \left(\frac{\mu(t)^2 + C}{\sigma_x^4(t)} + k^2 \right) \tag{29}$$

with C an integration constant. As before we can therefore see that the kinetic energy is proportional to $\sigma_x(t)^{-4}$. And so, as $\sigma_x(t)$ approaches 0 as $x \to R$, the kinetic energy of the spell approaches infinity. This intuitively makes sense to the authors, if we imagine a single element of the wave, as the probability approaches as a δ function, the element must also begin to move an infinitely large distance. This, we believe, is the fundamental flaw in the current setup of the magic theory and one which must be addressed. We will now detail potential solutions to this problem.

4 Proposed Solutions

There is one ***major*** solution we must consider: The previous section is wrong. *To step out of character for a moment, I am touching on some parts of physics I haven't in a long long while and so am myself quite rusty. This means that there's more than likely a few mistakes. For example, the elastic sheet equations may be totally incorrect, the fact they do not simplify to Hooke's law suggests some issue, though on review I think this may just be due to missing some of the components which would cancel out the 1/4 and return Hooke's law. I leave it in for transparency of my idiocy.* This is the preferred solution as it allows us to maintain the magic theory as is, with minimal alterations.

In this section we will discuss the two emerging sides of this argument: the Preservationists, who aim to preserve the magic theory as is with minor alterations, and the Aedificarists, who seek to rebuild the magic theory from the ground up. While there is no clear winner, we will attempt to summarise both arguments as best we can.

4.1 Preservationists

4.1.1 The Trivial Solution

One trivial solution to this is to simply say that these results are not a problem, i.e., the magic field can provide infinite energy to spells, and thus the problem cannot be ignored. However, this raises a few key problems. Chiefly, it goes against the evidence that we typically see light emitted from spells as they move, this implies that some energy is being lost as the spell travels. The light could, of course, be emitted by additional energy or another, as of yet, unkown mechanism, but this seems unlikely. Another issue, of course, is that infinite energy sources do not match any known natural phenomena and it seems unlikely that even the smallest spells have such a high cost.

4.1.2 High Energy Magic Field Collapse

This proposed solution suggests that perhaps our model is not entirely wrong, just shifted. Instead of the asymptopic behaviour occurring at a given range R, it instead occurs at some distance past R, and instead at R the spell reaches the maximum energy the magic field can hold, and, at $x > R$, the spell collapses. This would neatly solve the issue with minimal changes to the theory (simply replace R with $R + \Delta x$), but it does raise some important questions it cannot yet answer: What is the maximum allowed energy? Why is there a maximum allowed energy? Why is light emitted if energy increases?

4.1.3 Reformulating $\sigma_x(t)$

The asymptotic behaviour occurs because $\sigma_x(t) \to 0$, but there is no reason for this to be the case. Some researchers propose that by redefining $\sigma_x(t)$ we may preserve our behaviours. The fundamental flaw here is that energy is proportional to the inverse of $\sigma_x(t)$ in all cases ($E \propto \sigma_x(t)^{-n}$), but the theory demands that $\sigma_x(t) \to 0$, as $t \to t_R(x \to R)$. The proposed solution to this is define $\sigma_x(t)$ as :

$$\sigma'_x(t) = -\log\left(1 - \lambda_0\left(x - \frac{R}{v}\right)\right) \tag{30}$$

and then the standard deviation of the gaussian is the negative. This results in a negative potential energy which decreases to infinity rather than increasing. One note here though, is that this would also work with the negative of the standard definition of our width, and it is not clear that this value is at all physical. This is also only possible in the classical view, as the result has $E \propto \sigma_x(t)^{-3}$, and so the negative is preserved.

4.1.4 Non-standard Elastic Behvaiour in the Magic Field

One reason we see the infinite potential energy is because we model the magic field as an elastic sheet; however, there is no reason to believe that it is like this at all. This solution proposes that the magic field requires no energy to shift upwards, only to widen, and so the energy should be (though this is as of yet unverified) proportional to the width of the gaussian, hence preserving the original magic theory and corollaries. This may prove a subtle and interesting solution, but the fundamental theory work remains to be done.

4.2 Aedificarists

While fewer, there are a a number of Aedificarist (or newly built) theories which are gaining traction amongst certain academics. In this section we will give a very rough summary of some of their findings. The key issue these theories try to address is *why do spells have range in the first place?*. Unlike other physical phenomena, which disperses slowly over time, spells appear to have hard limits in which they cannot seem to exist beyond.

4.2.1 Variable Spell Velocity

In this theory, they alter the base very slightly, suggesting that spells maintain a constant standard deviation, and it is in-fact the speed which varies such that it hits zero at $x = R$ [7]. For example, one proposed model is parabolic spell velocity described by the general form:

$$v(x) = a(R^2 - x^2), \tag{31}$$

where a is some constant. There are, of course, other models, but this is the one proposed in [7]. Here we would see the kinetic energy of the spell decrease with time due to friction within the weave causing the escape of visible light. One issue this raises is that as the spell approaches R with $v \to 0$, it isn't clear that it will ever reach R within the common time frame we expect of spells. This also goes against commonly seen phenomena where spell speed appears constant. However, this may be a miscalculated "truth" from magic theorists, and while the parabolic model may be unlikely, there may be a more accurate model that gives this behaviour.

4.2.2 Energy Threshold Theory

One possibility of rectifying the Theory of Magic is to allow for dispersion in the wave (as we might expect in other physical phenomena, and thus conserving energy) but have the weave set a threshold on allowable energies [1]. The theory here states that we can follow standard dispersion relationships with spells but, at a critical point which we know as the range, the spell vanishes as the weave can no longer support a wave which is so spread out. While this does have some feasibility, more work is required to show that it matches phenomena seen.

4.2.3 Potential Wells

In other physical processes, we often see potential wells being a source of containig quantum and classical phenomena. For example, if we consider a physical well, a ball at the bottom of the well is free to move in the flat space at the bottom of the well, but to move outside the well requires a significant amount of energy input to the system to lift the ball out. This may well be the same as we see for spells [4], where they may freely propagate within some limits (the range), but to move beyond would require energy significantly higher than is available. This would prove interesting as it may allow for further prediction on phenomena like enchanting with standing wave patterns within smaller wells.

5 Conclusion

Through this brief review paper, we have sought to summarise some of the key flaws in the modern magic theory, the reasoning, and a brief survey of potential solutions. While there is no clear solution as of yet, there are two main types of theory, the preservationists, who aim to preserve as much of the original theory of magic as possible, and the aedificarists, who seek to recreate a new theory of magic on the ashes of the old one. The preservationist theories rely on modifying our model of the magic field itself, while the aedificarists often reformulate the entire structure of the theory. This conclusion will not specify the best of the theories put forward above, and only aims to summarise their findings and leaves preference as an exercise for the reader. Perhaps in the coming months we will see a true victor in this battle, however as it stands, all we know is that **The Theory is Dead, and we have killed it.**

References

[1] I. Bore. "Spell Dispersion Thresholds". In: *Standard Magic* (36ADR).

[2] The Gorilla of Desiny. "Arcane Engineering". In: *Magic in Artificery* (35ADR).

[3] The Gorilla of Destiny. "The Theory Of Magic". In: *University of Theoretical Magic Press* (32ADR).

[4] F. Foundson. "What's that Laddy? The spell fell down a potential well?: A proposed Aedioficarist Theory of Magic From Foundational Physics". In: *Physics in Magic* (36ADR).

[5] Joran H. "Analysis of Volume and Area Preference for Known Spells". In: *Magic in Artificery* (33ADR).

[6] I. Jor'ho. "Spontaneous Spell Casting in Nature Without Mortal Casting". In: *Magic in Nature* (30ADR).

[7] G. Lira. "A variational model of spell velocity". In: *Magic!* (36ADR).

[8] The Morrigan's Champion The Gorilla of Destiny Clarissa Cavendish. "Arcane Spectroscopy". In: *Astronomy and Astro-arcany* (34ADR).

A Normalising Classical Spells

Take the definition of A in eq. (5), and use it in the definition of $f(\delta, x)$ to get the probability density:

$$P(\delta, x) = \frac{1}{2\pi\sigma_D\sigma_x(t)} \times \exp -\frac{(\delta-\mu_D)^2}{2\sigma_D^2} \times \exp -\frac{(x-\mu_x(t))^2}{2\sigma_x(t)^2}. \tag{32}$$

To check this value of A, we must verify that the probability of getting the spell in any part of the real line x and and part of the damage domain δ is 1 (i.e., it exists somewhere). This probability is given by:

$$P = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} P(\delta, x)dxd\delta \tag{33}$$

Since the damage and spatial parts of $P(\delta, x)$ do not contain the other, we can seperate the integral as so:

$$P = \frac{1}{2\pi\sigma_D\sigma_x(t)} \int_{-\infty}^{\infty} \exp -\frac{(\delta-\mu_D)^2}{2\sigma_D^2} d\delta \int_{-\infty}^{\infty} \exp -\frac{(x-\mu_x(t))^2}{2\sigma_x(t)^2} dx \tag{34}$$

using the gaussian integral:

$$\int_{-\infty}^{\infty} e^{-ax^2} dx \tag{35}$$

we can say:

$$\int_{-\infty}^{\infty} \exp -\frac{(\delta-\mu_D)^2}{2\sigma_D^2} d\delta = \sqrt{\frac{\pi}{\left(\frac{1}{2\sigma_D^2}\right)}} \tag{36}$$

$$= \sqrt{2\pi\sigma_D^2} \tag{37}$$

and similarly:

$$\int_{-\infty}^{\infty} \exp -\frac{(x-\mu_x(t))^2}{2\sigma_x(t)^2} dx = \sqrt{\frac{\pi}{\left(\frac{1}{2\sigma_x(t)^2}\right)}} \tag{38}$$

$$= \sqrt{2\pi\sigma_x(t)^2}. \tag{39}$$

Therefore, eq. (34) becomes:

$$P = \frac{1}{2\pi\sigma_D\sigma_x(t)} \times \sqrt{2\pi\sigma_D^2} \times \sqrt{2\pi\sigma_x(t)^2}. \tag{40}$$

$$\Rightarrow P = 1 \tag{41}$$

as expected, verifying eq. (5).

B Why $\exp i(kx - \omega t)$ is a Wave

I was going to write here but this is one of those times where I have to just admit defeat and say "go watch someone better than me". I may add to this in the future but for the sake of time I instead point you towards three blue one brown's video on it: https://www.youtube.com/watch?v=-j8PzkZ70Lg . Frankly, this isn't super relevant to what is presented here, but I understand seeing waves as $\exp i(kx - \omega t)$ can be a bit unusual if you aren't used to it. Part of the reason we can know this is true is from Euler's formula:

$$e^{i\theta} = \cos(\theta) + i\sin(\theta) \tag{42}$$

in the complex plane, this traces out a unit circle that undergoes one rotation after θ (in radians) changes by 2π. If we were to take only the real part of this equation, we can see that we are just returned $\cos(\theta)$, which we all know is a wave. In our case that gives $\cos(kx - \omega t)$. You might ask why we then choose to use the exponential form instead, and, in short, it's because a *lot* of maths gets a lot easier when we are dealing with e (as this paper actually shows, trust me, the integrations are bad enough without having to start introducing cosines). There are probably better and more nuanced reasons I am forgetting, feel free to remind me of them.

C Deriving Elastic Sheet Potential Energy

Potential energy of a continuum is given by:

$$U = \frac{1}{2} C_{ijkl} \epsilon_{ij} \epsilon_{kl} \tag{43}$$

using Einstein summation notation with i, j, k, l indices equal to x, δ, or z. C is defined:

$$C_{ijkl} = \lambda \delta_{ij} \delta_{kl} + \mu(\delta_{ij} \delta_{jl} + \delta_{il} \delta jk), \tag{44}$$

with δ_{ij} being Kronecker deltas. Similarly. we define $\epsilon_{ij} = \frac{1}{2}(\partial_i u_j + \partial_j u_i)$. Assuming x and δ are independent this gives:

$$U = \frac{1}{8} \mu \left((\partial_x g)^2 + (\partial_\delta f)^2 \right) \tag{45}$$

Over the entire elastic space, we can then define the total energy as:

$$U_{\text{total}} = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} U dx d\delta \tag{46}$$

Author's Note: This is an area of physics I'm not super well versed in so if you're hunting for mistakes, this may well be the place.